1. Use the differentiation formulæ to find the derivatives of the functions below.

(a) (2 pts) $f(x) = 3\sqrt{x} + \dfrac{1}{x^2}$; $\quad f'(x) = \dfrac{d}{dx}\left(3x^{1/2} + x^{-2}\right) = 3 \cdot \left(\dfrac{1}{2}x^{-1/2}\right) + (-2)x^{-3} = \dfrac{3}{2}x^{-1/2} - 2x^{-3}$.

(b) (2 pts) $v = (4t^2+3)(t^3+2t+1)$;

$$\frac{dv}{dt} = (4t^2+3)'(t^3+2t+1) + (4t^2+3)(t^3+2t+1)'$$
$$= 8t(t^3+2t+1) + (4t^2+3)(3t^2+2) \qquad \left(= 20t^4 + 33t^2 + 8t + 6\right)$$

using the product rule...

... or

$$\frac{dv}{dt} = \frac{d}{dt}\left(4t^5 + 11t^3 + 4t^2 + 6t + 3\right)$$
$$= 20t^4 + 33t^2 + 8t + 6$$

not using the product rule.

(c) (2 pts) $y = \dfrac{2x^2+3}{5x+1}$; $\quad y' = \dfrac{(2x^2+3)'(5x+1) - (2x^2+3)(5x+1)'}{(5x+1)^2} = \dfrac{4x(5x+1) - 5(2x^2+3)}{(5x+1)^2}$

$$\left(= \frac{10x^2 + 4x - 25}{(5x+1)^2}\right)$$

2. (7 pts) Use the ***definition*** of the derivative to find the slope of the curve $y = \dfrac{3}{x}$ at the point $x_0 = 2$.

$$\left.\frac{dy}{dx}\right|_{x=2} = \lim_{h\to 0} \frac{\dfrac{3}{2+h} - \dfrac{3}{2}}{h} = \lim_{h\to 0} \frac{\dfrac{3\cdot 2 - 3(2+h)}{2(2+h)}}{h} = \lim_{h\to 0} \frac{-3\cancel{h}}{2\cancel{h}(2+h)} = \lim_{h\to 0} \frac{-3}{2(2+h)} = -\frac{3}{4}.$$

3. (7 pts) A firm's ***marginal cost*** function is given by

$$\frac{dc}{dq} = \sqrt{3q^2 + 16},$$

where cost (c) is measured in \$1000s and output (q) is measured in 100s of units. By approximately how much will the firm's cost increase if its output increases from 400 units to 430 units?

When output is 400 units, $q = 4$, and increasing output to 430 units means that $\Delta q = 0.3$ (because q is measured in 100 s of units). Now we use linear approximation:

$$\Delta c \approx \left.\frac{dc}{dq}\right|_{q=4} \cdot \Delta q = \left.\sqrt{3\cdot q^2 + 16}\right|_{q=4} \cdot (0.3) = \sqrt{64}\cdot(0.3) = 2.4,$$

which means that cost will increase by about \$2400.00 (because c is measured in \$1000 s).