1. Find the derivatives (using the formulas, not the definition).
Clean up your answers.

(a) (3 pts) $y = 3x\ln(x^2+1)$; $\dfrac{dy}{dx} = 3\ln(x^2+1) + 3x\cdot\dfrac{1}{x^2+1}\cdot 2x = 3\ln(x^2+1) + \dfrac{6x^2}{x^2+1}$.

Product rule, and chain rule for $\ln(x^2+1)$.

(b) (3 pts) $f(t) = 10e^{t^2+4}$; $f'(t) = 10e^{t^2+4}\cdot(2t) = 20te^{t^2+4}$.

Chain rule.

2. (6 pts) Find the ***quadratic*** Taylor polynomial for the function $f(x) = \sqrt{x}$ centered at $x_0 = 25$, and use this polynomial to approximate $\sqrt{24.5}$.

Derivatives: $f'(x) = \frac{1}{2}x^{-1/2}$, $f''(x) = -\frac{1}{4}x^{-3/2}$.

Evaluate at x_0*:* $f(25) = 5$, $f'(25) = \dfrac{1}{10}$, $f''(25) = -\dfrac{1}{500}$.

Taylor polynomial:

$$\begin{aligned} T_2(x) &= f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2}(x-x_0)^2 \\ &= 5 + \frac{1}{10}(x-25) - \frac{1}{1000}(x-25)^2. \end{aligned}$$

Approximate $\sqrt{24.5}$*:*

$$\sqrt{24.5} = f(24.5) \approx T_2(24.5) = 5 + \frac{1}{10}(-1/2) - \frac{1}{1000}(-1/2)^2 = 4.94975.$$

3. The demand equation for a monopolist's product is $q = 10\sqrt{400-0.8p}$

(a) (4 pts) Find the price-elasticity of demand for this good ***as a function of the price*** p.

$$\eta_{q/p} = \frac{dq}{dp}\cdot\frac{p}{q} = \overbrace{10\cdot\frac{1}{2}\cdot(400-0.8p)^{-1/2}\cdot(-0.8)}^{dq/dp}\cdot\frac{p}{\underbrace{10\sqrt{400-0.8p}}_{q}} = -\frac{0.4p}{400-0.8p}$$

(b) (2 pt) What is the price elasticity of demand when $p = 200$?

$$\eta_{q/p}\Big|_{p=200} = -\frac{80}{240} = -\frac{1}{3}.$$

(c) (2 pts) Use your answer to (b) to find the firm's *marginal revenue* when $p = 200$.

$$\frac{dr}{dq} = p\left(1 + \frac{1}{\eta_{q/p}}\right) \implies \frac{dr}{dq}\Big|_{p=200} = 200\left(1 + \frac{1}{-1/3}\right) = -400.$$