1. (8 pts) Find the absolute minimum and absolute maximum values of the function

$$g(u) = 2u^3 - 21u^2 + 60u + 1$$

in the interval $[0, 6]$.
Step 1. *Find critical points...*

$$g'(u) = 6u^2 - 42u + 60 = 6(u^2 - 7u + 10) = 6(u-2)(u-5),$$

so $g'(u) = 0 \implies u = 2$ or $u = 5$.
Step 2. Evaluate $g(u)$ at the endpoints (0 and 6) of the interval and at the critical points (2 and 5). The largest value will be the absolute maximum and the smallest value will be the absolute minimum.

u	$g(u)$	
0	$g(0) = 1$	$\Leftarrow$ absolute minimum
2	$g(2) = 53$	$\Leftarrow$ absolute maximum
5	$g(5) = 26$	
6	$g(6) = 37$	

2. (8 pts) Find the critical point(s) and critical value(s) of the function $f(x) = 4x^2e^{-0.5x}$, and use the *first derivative test* to classify the critical value(s) as relative minima, relative maxima or neither.

Step 1. *Find critical points and critical values...*

$$f'(x) = 8xe^{-0.5x} + 4x^2e^{-0.5x} \cdot (-0.5) = 8xe^{-0.5x} - 2x^2e^{-0.5x} = 2xe^{-0.5x}(4-x),$$

so $f'(x) = 0 \implies x = 0$ or $x = 4$.

I.e., the critical points are $x_1 = 0$ and $x_2 = 4$, and the critical values are $f(0) = 0$ and $f(4) = 64e^{-2} \approx 8.66$.

Step 2. *First derivative test...* Evaluate the sign of $f'(x)$ on either side of each critical point.

For $x_1 = 0$, we have $f'(-1) = -2 \cdot e^{0.5}(4+1) < 0$ and $f'(1) = 2 \cdot e^{-0.5}(4-1) > 0$, so $f(0) = 0$ is a relative ***minimum*** value.†

For $x_2 = 4$, we have $f'(1) = 2 \cdot e^{-0.5}(4-1) > 0$ and $f'(5) = 10e^{-2.5}(4-5) < 0$, so $f(5) \approx 8.66$ is a relative ***maximum*** value.

3. (4 pts) Does the function $f(x)$, above, have an absolute maximum value in the interval $(0, \infty)$? Justify your answer.

Yes, $f(4) \approx 8.66$ is the absolute maximum value of $f(x) = 4x^2e^{-0.5x}$ in $(0, \infty)$ because (i) $f(4)$ is a *relative* maximum value and (ii) there is only one critical point in $(0, \infty)$.

†In fact, $f(0) = 0$, is the ***absolute minimum*** value of $f(x)$ in $(-\infty, \infty)$. Can you say why?