Linear approximation for percentage change.

By definition,

$$\eta_{y/x} = \lim_{\Delta x \to 0} \frac{\%\Delta y}{\%\Delta x}$$

This means: if Δx is small, then

$$\eta_{y/x} \approx \frac{\%\Delta y}{\%\Delta x}$$

and therefore, if Δx is small, then

$$\%\Delta y \approx \eta_{y/x} \cdot (\%\Delta x)\,.$$

Example: If $y = 2x^{0.4}$, then we saw that $\eta_{y/x} = 0.4$ (for all x). What is the approximate percentage change in y if x increases from 10 to 10.3?

The percentage change in x is $\%\Delta x = (0.3/10) \cdot 100\% = 3\%$,

and so $\quad \%\Delta y \approx \eta_{y/x} \cdot \%\Delta x = 0.4 \cdot 3\% = 1.2\%$.

(The actual percentage change in y is $\frac{10.3^{0.4} - 10^{0.4}}{10^{0.4}} \cdot 100\% \approx 1.19\%$)

Classifying price-elasticity of demand.

For a normal good, the price elasticity of demand (as defined here and in our textbook) is always negative, because

$$\eta_{q/p} = \frac{dq}{dp} \cdot \frac{p}{q} \quad \text{or} \quad \eta_{q/p} = \frac{p/q}{dp/dq}$$

and p and q are both ***positive***, while both dq/dp and dp/dq are ***negative***.

This makes sense because for a normal good, if the price rises then the demand will go down and vice versa, so if $\%\Delta p > 0$, we expect $\%\Delta q < 0$ and vice versa.

Definition:

- If $\eta_{q/p} < -1$ (so $|\eta_{q/p}| > 1$), demand is said to be ***elastic***.
- If $0 > \eta_{q/p} > -1$ (so $|\eta_{q/p}| < 1$), demand is said to be ***inelastic***.
- If $\eta_{q/p} = -1$ (so $|\eta_{q/p}| = 1$), demand is said to have ***unit elasticity*** (or to be unit-elastic).

When demand is *elastic* a 1% percentage change in price will result in larger than 1% change in demand (in the opposite direction).

When demand is *inelastic*, a 1% percentage change in price will result in smaller than 1% change in demand (again, in the opposite direction).

Intuition: If demand is *elastic*, then *lowering* the price of a good will result in higher revenue. And if the demand is *inelastic*, then *raising* the price of a good will result in higher revenue.

To verify this intuition, we study the relation between *price-elasticity of demand* and *marginal revenue.* Using the product rule, we see that

$$\frac{dr}{dq} = \frac{d}{dq}\overbrace{(pq)}^{r} = \frac{dp}{dq}\cdot q + p\cdot\frac{dq}{dq} = p + q\frac{dp}{dq} = p\left(1 + \frac{q}{p}\cdot\frac{dp}{dq}\right)$$

But

$$\frac{q}{p}\cdot\frac{dp}{dq} = \frac{1}{\frac{p}{q}}\cdot\frac{dp}{dq} = \frac{\frac{dp}{dq}}{\frac{p}{q}} = \frac{1}{\eta} \implies \boxed{\frac{dr}{dq} = p\left(1 + \frac{1}{\eta}\right)}$$

Therefore:

1. If $\eta < -1$ (elastic demand), then $0 > \frac{1}{\eta} > -1$ so $\dfrac{dr}{dq} = p\left(1 + \frac{1}{\eta}\right) > 0$. In this case, *lowering* p will *raise* q and since $dr/dq > 0$, this will *increase revenue.*

2. If $0 > \eta > -1$ (inelastic demand), then $\frac{1}{\eta} < -1$ so $\dfrac{dr}{dq} = p\left(1 + \frac{1}{\eta}\right) < 0$. In this case, *raising* p will *lower* q and since $dr/dq < 0$, this will *increase* revenue.